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## Mathematics Instruction 6



## Lesson 29 Part 1: Introduction 85 Analyze Numerical Data

You've learned how to measure the center of data values with median and mean. Take a look at this problem.

Death Valley National Park in the western United States is known for its extreme temperatures. This table shows high temperatures for the first 15 days of October.

| $99^{\circ} \mathrm{F}$ | $113^{\circ} \mathrm{F}$ | $99^{\circ} \mathrm{F}$ | $97^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ |
| :---: | :---: | :---: | :---: | :---: |
| $88^{\circ} \mathrm{F}$ | $88^{\circ} \mathrm{F}$ | $90^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $81^{\circ} \mathrm{F}$ |
| $71^{\circ} \mathrm{F}$ | $80^{\circ} \mathrm{F}$ | $79^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $96^{\circ} \mathrm{F}$ |

Use median and mean to describe the data.

## Q Explore It

## Use the math you already know to solve this problem.

Construct a dot plot.
Death Valley National Park High Temperatures


Describe the shape of the data. What does the shape tell you about the temperatures?
$\qquad$
$\qquad$
Do you notice any outliers? Explain. $\qquad$
$\qquad$
What is the median temperature? $\qquad$ mean? $\qquad$

- What's similar about the mean and median? What's different? Explain. $\qquad$
$\qquad$
$\qquad$


## Q Find Out More

What would happen to the median and mean if you eliminate the outlier and replace it with a less extreme temperature, like $100^{\circ} \mathrm{F}$ ?

The data set and dot plot would look like this:

| $99^{\circ} \mathrm{F}$ | $100^{\circ} \mathrm{F}$ | $99^{\circ} \mathrm{F}$ | $97^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ |
| ---: | ---: | ---: | ---: | ---: |
| $88^{\circ} \mathrm{F}$ | $88^{\circ} \mathrm{F}$ | $90^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $81^{\circ} \mathrm{F}$ |
| $71^{\circ} \mathrm{F}$ | $80^{\circ} \mathrm{F}$ | $79^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $96^{\circ} \mathrm{F}$ |

Death Valley National Park High Temperatures


The median does not change; it is still $88^{\circ} \mathrm{F}$. The mean changes from about $89.3^{\circ} \mathrm{F}$ to about $88.5^{\circ} \mathrm{F}$. In this context, the outlier influences the mean but not the median.

## Reflect

1 Explain why outliers affect the mean.

## Read the problem below. Then explore how the interquartile range (IQR) measures variability.

Consider the first set of Death Valley National Park temperatures that you saw. What does the interquartile range ( IQR ) tell you about the variability of the temperatures?

| $99^{\circ} \mathrm{F}$ | $113^{\circ} \mathrm{F}$ | $99^{\circ} \mathrm{F}$ | $97^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ |
| ---: | ---: | ---: | ---: | :--- |
| $88^{\circ} \mathrm{F}$ | $88^{\circ} \mathrm{F}$ | $90^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $81^{\circ} \mathrm{F}$ |
| $71^{\circ} \mathrm{F}$ | $80^{\circ} \mathrm{F}$ | $79^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $96^{\circ} \mathrm{F}$ |

## Q. Model It

You can find the quartile values to understand the problem.


## Q. Model It

## You can draw a box plot to understand the problem.

Death Valley National Park High Temperatures


## Connect It

## Now you will solve the problem using the models.

2 Calculate the IQR. What does it mean within this context?
$\qquad$
$\qquad$
3 Look at the box plot. How many data points are represented by the box? What does this box mean?
$\qquad$
$\qquad$
4 If you replace the outlier $\left(113^{\circ} \mathrm{F}\right)$ with $100^{\circ} \mathrm{F}$, what happens to the IQR? What happens to the range? Explain.
$\qquad$
$\qquad$
$\qquad$
5 Within this context, explain what the median and the IQR tell you about the data.
$\qquad$
$\qquad$

## Try It

Use what you just learned about median and IQR to solve this problem. Show your work on a separate sheet of paper.

6 Are the median and IQR typically affected by outliers? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Read the problem below. Then explore how the Mean Absolute Deviation (MAD) measures variability.

Consider another way to describe the Death Valley National Park temperature data. Calculate the mean absolute deviation (MAD). What does the MAD tell you about the variability of the temperatures?

| $99^{\circ} \mathrm{F}$ | $113^{\circ} \mathrm{F}$ | $99^{\circ} \mathrm{F}$ | $97^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ |
| ---: | ---: | ---: | ---: | :--- |
| $88^{\circ} \mathrm{F}$ | $88^{\circ} \mathrm{F}$ | $90^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $81^{\circ} \mathrm{F}$ |
| $71^{\circ} \mathrm{F}$ | $80^{\circ} \mathrm{F}$ | $79^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $96^{\circ} \mathrm{F}$ |

## Q. Model It

You can make a table to understand the problem.

| Data Value | Deviation from Mean <br> Mean $=89.3^{\circ} \mathrm{F}$ | Absolute Deviation |
| :---: | :---: | :---: |
| $99^{\circ} \mathrm{F}$ | 9.7 | 9.7 |
| $88^{\circ} \mathrm{F}$ | -1.3 | 1.3 |
| $71^{\circ} \mathrm{F}$ | -18.3 | 18.3 |
| $113^{\circ} \mathrm{F}$ | 23.7 | 23.7 |
| $88^{\circ} \mathrm{F}$ | -1.3 | 1.3 |
| $80^{\circ} \mathrm{F}$ | -9.3 | 9.3 |
| $99^{\circ} \mathrm{F}$ | 9.7 | 9.7 |
| $90^{\circ} \mathrm{F}$ | 0.7 | 0.7 |
| $79^{\circ} \mathrm{F}$ | -10.3 | 10.3 |
| $97^{\circ} \mathrm{F}$ | 7.7 | 7.7 |
| $84^{\circ} \mathrm{F}$ | -5.3 | 5.3 |
| $84^{\circ} \mathrm{F}$ | -5.3 | 5.3 |
| $91^{\circ} \mathrm{F}$ | 1.7 | 1.7 |
| $81^{\circ} \mathrm{F}$ | -8.3 | 8.3 |
| $96^{\circ} \mathrm{F}$ | 6.7 | 6.7 |
|  |  | MAD: $\frac{119.3}{15}=7.95$ |

## Connect It

## Now you will solve the problem using the model.

7 What does the MAD mean within this context?

8 Given this context, does the MAD value indicate a high or low variability?
$\qquad$
$\qquad$
$\qquad$

9 If you replace the outlier $\left(113^{\circ} \mathrm{F}\right)$ with $100^{\circ}$, what happens to the MAD? Explain.

10 Within this context, how are the mean and the MAD different? How are they related?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Try It

Use what you just learned about mean and MAD to answer this question.
11 Are the mean and MAD typically affected by outliers? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

The student recognized how an outlier can affect a measure of center.


## Pair/Share

How could you justify your answer with a graph of the data?

Does the outlier affect the mean or the median?


## B Pair/Share

Given this context, why does it make sense to measure the center without the outlier?

## Study the student model below. Then solve problems 12-14.

## Student Model

Zoe and her lab partners have a mystery mineral that they are trying to identify. They take turns finding the mass of the same mineral before finding its density. Here are their measurements:

$$
50.1 \mathrm{~g}, 50.4 \mathrm{~g}, 20.5 \mathrm{~g}, 50.2 \mathrm{~g}, 50.2 \mathrm{~g}
$$

Given this context and these data points, is the median or the mean a better measure of center?

Look at how you could show your work.

```
20.5 50.1 50.2 50.2 50.4
```

The median, $\mathbf{5 0 . 2}$, is a better measure of data because all of the masses except one are around $\mathbf{5 0} \mathbf{~ g}$. Finding the mean with an outlier such as 20.5 would produce a number much lower than the mineral's actual mass.

12 Use the data set in the student model above to calculate the mean and the median without the outlier. Now which is a better measure of center? Explain.
Show your work.

Solution: $\qquad$

13 Complete the table below. How many data points are represented in this histogram?

Reaction Time of Teen Drivers


| Intervals (seconds) | Number of Drivers |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  | TOTAL $=$ |

14 Look at the histogram in problem 13. Which of the following statements do you know to be TRUE?
A The data are skewed.
B The median is 0.55 seconds.
C The range is 0.7 seconds.
D The center of the data is between 0.5 to 0.6 seconds.
Jo chose B as the correct answer. How did she get that answer?
$\qquad$
$\qquad$

How many data points are in each interval?

## OPair/Share

In what interval are most of the data values clustered? Does this make sense given the context?

How do histograms show center and variability?


## OPair/Share

How can you help Jo understand her error?

## Solve the problems.

1 Look at the box plots. Which is a TRUE statement?

## Average Time Spent on Homework



A If you moved the students from the top $25 \%$ in Class 1 into Class 2, the Class 2 median would increase.

B If you omitted the student(s) who did no homework in Class 1, the IQR for Class 1 would increase.

C If you combined the data from both classes onto one box plot, the range would be 105.
D If you combined the data from both classes onto one box plot, the median would be double the current median for Class 1.

2 The cumulative frequency histogram represents the number of minutes per school week that

NUMBER OF MINUTES STUDENTS WATCH TV 320 students watch television.


What fraction of the population watches 150 minutes or less of television?


3 This table represents daily attendance at 2 movie theaters for one week.

|  | Theater 1 | Theater 2 |
| :--- | :---: | :---: |
| Monday | 42 | 24 |
| Tuesday | 50 | 28 |
| Wednesday | 48 | 20 |
| Thursday | 60 | 92 |
| Friday | 80 | 88 |
| Saturday | 212 | 95 |
| Sunday | 65 | 90 |

## Part A

Calculate the mean and median for each theater's attendance. Round your answers to the nearest whole number.

Theater 1: Mean $\qquad$ Median $\qquad$
Theater 2: Mean $\qquad$ Median $\qquad$

## Part B

Which is a better measure of center for Theater 1, mean or median? Explain.
$\qquad$
$\qquad$
$\qquad$

## Part C

Which is a better measure of center for Theater 2, mean or median? Explain.
$\qquad$
$\qquad$

Self Check Go back and see what you can check off on the Self Check on page 265.

## T1 Ready Common Core


and

## Lesson 29 (Sudent Book pages 296 -305)

Analyze Numerical Data

## LESSON OBJECTIVES

- Interpret a set of numerical data by noticing and describing patterns and deviations.
- Understand mean absolute deviation (MAD).
- Determine variability (IQR, MAD).


## PREREQUISITE SKILLS

- Find the mean and median of a set of data.
- Describe the range, spread, and outliers of a data set.
- Graph data with dot plots, histograms, and box plots.
- Understand absolute value.


## VOCABULARY

There is no new vocabulary.

## THE LEARNING PROGRESSION

An understanding of data analysis is important in today's world. Employees must understand and use data in many careers. We use data to make choices as we buy products and make healthcare decisions.
In Grade 6, students learn to find measures of center and variability. They also expand the types of data representations they use beyond the picture and bar graphs they learned in Grades 3 through 5.
In this lesson, students use dot plots, histograms, and box plots to examine the spread of data and the effect outliers have on the mean and median. They also find the mean absolute deviation (MAD), a measure of variability.
In later grades, students continue to analyze data using measures of central tendency and variability. They will use various methods to display data effectively. As students extend their study of statistics, they will build on the skills and concepts learned in Grade 6.

## TIT Ready Teacher Toolbox

Teacher-Toolbox.com

|  | $\left\|\begin{array}{c} \text { Prerequisite } \\ \text { Skills } \end{array}\right\|$ | 6.SP.B.5a, 6.SP.B.5b 6.SPB.5c, 6.SP.B.5d |
| :---: | :---: | :---: |
| Ready Lessons | $\checkmark$ | $\checkmark$ |
| Tools for Instruction | $\checkmark$ | $\checkmark$ |
| Interactive Tutorials | $\checkmark \checkmark$ |  |

## CCSS Focus

6.SP.B.5 Summarize numerical data sets in relation to their context, such as by:
a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
ADDITIONAL STANDARDS: 6.SP.A.2, 6.SP.A.3, 6.SP.B. 4 (see page A42 for full text)
STANDARDS FOR MATHEMATICAL PRACTICE: SMP 2-5 (see page A9 for full text)

## AT A GLANCE

Students describe a set of data using the mean and median.

## STEP BY STEP

- Tell students this page models using the mean and median to describe data.
- Have students read the problem at the top of the page.
- Work through Explore It as a class.
- Have a volunteer explain how to make a dot plot. Stress the need for accuracy.
- Have students describe the spread, then suggest what the spread might mean in the context of the problem.
- Ask student pairs or groups to explain their answers for finding the outliers, the median, and the mean.
- As students compare and contrast the mean and the median, have them explain how the shape of the data illustrates the similarities and differences between the two measures of central tendency.


## ELL Support

- Call 7 students to line up in the front of the class.
- Ask the class to name the student in the center. Say that center is another word for "middle."
- Ask the students to stay in the same order, but to spread out. Discuss the meaning of spread.
- Have all the students move close together except for one on an end who should move away from the others. Explain that this student is an outlier.

SMP Tip: Students can use dot plots to help them understand a set of data. Encourage them to use dot plots and other appropriate representations as tools for visualizing data and drawing conclusions about them (SMP 5).

Lesson 29 Part 1: Introduction 28
Analyze Numerical Data

You've learned how to measure the center of data values with median and mean.
Take a look at this problem.
Death Valley National Park in the western United States is known for its extreme temperatures. This table shows high temperatures for the first 15 days of October.

| $99^{\circ} \mathrm{F}$ | $113^{\circ} \mathrm{F}$ | $99^{\circ} \mathrm{F}$ | $97^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ |
| :---: | :---: | :---: | :---: | :---: |
| $88^{\circ} \mathrm{F}$ | $88^{\circ} \mathrm{F}$ | $90^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $81^{\circ} \mathrm{F}$ |
| $71^{\circ} \mathrm{F}$ | $80^{\circ} \mathrm{F}$ | $79^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $96^{\circ} \mathrm{F}$ |

Use median and mean to describe the data

Q Explore It
Use the math you already know to solve this problem.

- Construct a dot plot.

Death Valley National Park High Temperatures
$\underset{70}{\stackrel{0}{0}}$

- Describe the shape of the data. What does the shape tell you about the temperatures? The data is spread out symmetrically, and most temperatures are between $\mathbf{8 0 - 1 0 0}$ degrees. There is not one temperature that occurs frequently.
- Do you notice any outliers? Explain. $\underline{113^{\circ} \mathrm{F} \text { is an outlier because it is } 13 \text { degrees above }}$ where most of the temperatures are. You could also consider that $71^{\circ} \mathrm{F}$ is an outlier.
- What is the median temperature? $88^{\circ} \mathrm{F}$ mean? $89.3^{\circ} \mathrm{F}$
- What's similar about the mean and median? What's different? Explain. The mean and median temperatures are similar because the data is symmetrically spread out. The mean is slightly greater than the median because there is a high outlier at $113^{\circ} \mathrm{F}$.

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## Mathematical Discourse

- Why is it often helpful to make a dot plot as a first step in analyzing data?

Students might say it is a quick way to order the numbers. They might also say that it is a good way to study the spread and look for clusters of numbers.

- Would you rather use the table or the dot plot to find the median? How about the mean? Explain why.

Students' answers will vary. Many might say the dot plot is easiest to use for finding the median because you can count points to see which number is in the middle. They might also say that the table is easiest for the mean because it doesn't matter what order the numbers are in when you add them. Be sure all points of view are explained.

## AT A GLANCE

Students examine the effect of an outlier on the mean and median of a set of data.

## STEP BY STEP

- Read Find Out More as a class.
- Have a volunteer explain how the data set on page 297 is different from the data on page 296.
- Have students compare the dot plots on pages 296 and 297 and then predict how the change of one temperature will affect the mean and the median values.
- Based on student answers to the Reflect directive, discuss why the mean changed but the median stayed the same.

SMP Tip: Students learn to reason abstractly and quantitatively as they compare sets of data with and without outliers (SMP 2). As you work through examples, have students predict the shape of the data before they actually graph it. Then discuss how close their predictions were to the actual results.


## Real-World Connection

In many jobs, people use information from what happened in the past to make decisions about what they will do in the future. What are some of these jobs? What types of decisions are affected by past numbers?

Examples: An event planner deciding how much food to buy; a medical researcher deciding if a new medicine is curing people; a school superintendent deciding how many teachers are needed at each school; a traffic planner deciding where new stoplights are needed; a coach deciding which players are the top scorers or defenders.

## AT A GLANCE

Students use models to help them visualize how the interquartile range describes the variability of a set of data.

## STEP BY STEP

- Read the problem at the top of the page as a class.
- Review the meaning of interquartile range (IQR).
- Talk about how to use an ordered list of the data to show the interquartile range.
- Discuss how to draw a box plot to show the interquartile range.


## Hands-On Activity

Find the quartiles and interquartile range of a set of numbers.

Materials: playing cards without the face cards.

- Give each student 12 cards. Each student should put the cards in ascending order and then divide them into 4 groups: the lowest fourth, the second fourth, the third fourth, and the highest fourth.
- Model the term quartiles for the groups. Call on students to describe the cards in each of their quartiles.
- Ask students to focus on the two middle groups. Note that those cards are one-half of all their cards.
- Have students find the card representing the lower quartile and the card representing the upper quartile. Have them find the difference of the two cards. Tell them the difference is called the interquartile range, or $I Q R$ for short.
- Redistribute the cards so that each student has a different set of 12 . Have them find the quartiles and interquartile range independently. Have students pair up and check each others' work.
© Part 2:Modeded hnstuction
Lesson 29

Read the problem below. Then explore how the interquartile range (IQR) measures variability.

Consider the first set of Death Valley National Park temperatures that you saw. What does the interquartile range (IQR) tell you about the variability of the temperatures?

| $99^{\circ} \mathrm{F}$ | $113^{\circ} \mathrm{F}$ | $99^{\circ} \mathrm{F}$ | $97^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ |
| :---: | :---: | :---: | :---: | :---: |
| $88^{\circ} \mathrm{F}$ | $88^{\circ} \mathrm{F}$ | $90^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $81^{\circ} \mathrm{F}$ |
| $71^{\circ} \mathrm{F}$ | $80^{\circ} \mathrm{F}$ | $79^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $96^{\circ} \mathrm{F}$ |

Q Model It
You can find the quartile values to understand the problem.


Model It
You can draw a box plot to understand the problem.


## Mathematical Discourse

- What are the five important points on a box plot? What does each point tell you?

Students should name the minimum, the lower quartile, the mean, the upper quartile, and the maximum. They should explain what each value means relative to the data. Listen for phrases such as "the lowest $25 \%$," "the middle temperature," and "the highest temperature."

- Do you only have five points on a box plot? Do the other points matter? Explain why or why not.

Students should explain that the other points do matter because they determine the five key points. Those five points are spread out through the quartiles, with $25 \%$ of all points in each quartile.

## AT A GLANCE

Students revisit the problem on page 298 and use a box plot to analyze the data.

## STEP BY STEP

- Read Connect It as a class. Be sure to point out that the questions refer to the problem on page 298.
- As you discuss each question, ask students to restate responses given by other students in their own words. Allow time for clarifying questions.
- Review the significance of the interquartile range (IQR) in the context of the data.
- Discuss why replacing the outlier affects the range but not the median or interquartile range.
- Have students consider when they would be most interested in the interquartile range compared to the range.
- Have students share their responses to problem 5. Use their responses to increase classroom understanding about what the median and the IQR indicate about the entire data set.

SMP Tip: When students explain what various statistical measures mean and predict how changes in the data affect those statistical measures, students must construct viable arguments and critique the reasoning of others (SMP 3). Encourage students to help each other express their ideas clearly, using proper mathematical terminology.

8 Part 2: Guided Instruction Lesson 29

## Q Connect it

Now you will solve the problem using the models.
2 Calculate the IQR. What does it mean within this context?
$\mathrm{IQR}=97^{\circ} \mathrm{F}-81^{\circ} \mathrm{F}=16^{\circ} \mathrm{F}$. This means the middle $50 \%$ of temperatures are within a 16 degree range.
3 Look at the box plot. How many data points are represented by the box? What does this box mean?
There are 9 data points in this box. The box shows that half the temperatures were between $81^{\circ} \mathrm{F}$ and $97^{\circ} \mathrm{F}$. It also shows the IQR.

4 If you replace the outlier $\left(113^{\circ} \mathrm{F}\right)$ with $100^{\circ} \mathrm{F}$, what happens to the IQR? What happens to the range? Explain.
The IQR will not change because the Q1 and Q3 values are not affected by an
outlier. The range changes from 42 degrees to 29 because $113-71=42$ and
$100-71=29$.
5 Within this context, explain what the median and the IQR tell you about the data. The median, $88^{\circ} \mathrm{F}$, says $50 \%$ of the temperatures are higher and $50 \%$ lower. The IQR tells the range of the middle $50 \%$ of temperatures recorded.

Use what you just learned about median and IQR to solve this problem. Show your work on a separate sheet of paper.
6 Are the median and IQR typically affected by outliers? Explain.
No. Changing the high outlier to a value closer to the middle $50 \%$ of data points did not change the median or IQR.

## TRY IT SOLUTION

6 Solution: No. Changing the high outlier to a value closer to the middle $50 \%$ of data points did not change the median or IQR.

ERROR ALERT: Students who answered "yes" may have confused median and IQR with mean and range. Outliers affect both the mean and the range of a data set.

## AT A GLANCE

Students see how to analyze data from the problem on page 296 using the mean absolute deviation.

## STEP BY STEP

- Read the problem at the top of the page as a class.
- Say that the mean absolute deviation (MAD) tells how much the data varies from the mean. It is a way to describe how spread-out the data are. Remind students that the mean for the data set shown is 89.3.
- Study each column in the table. Have students explain how to find the deviation from the mean.
- Have students recall the concept of absolute value as you explain that the absolute deviation is the distance each value is from the mean regardless of whether that value is above or below the mean.
- Have students explain how to use the absolute deviations to find the mean absolute deviation.


## Visual Model

## Illustrate variability using line plots.

- Sketch two line plots on the board. Each number line should extend from 20 through 30.
- On the first number line, put one dot over the 23, two dots over the 24 , five over the 25 , two over the 26 , and one over the 27 .
- On the second number line, put one dot over each number.
- Have students discuss how the line plots are the same. Note that they are both symmetrical and centered around 25 , so the mean and median would be 25 for both sets of data.
- Ask students to explain how the line plots look different. Tie the idea of a greater spread to greater variability or deviation from the center.
© Part 3:Modeded hnstuction Lesson 29

Read the problem below. Then explore how the Mean Absolute Deviation (MAD) measures variability.

Consider another way to describe the Death Valley National Park temperature data. Calculate the mean absolute deviation (MAD). What does the MAD tell you about the variability of the temperatures?

| $99^{\circ} \mathrm{F}$ | $113^{\circ} \mathrm{F}$ | $99^{\circ} \mathrm{F}$ | $97^{\circ} \mathrm{F}$ | $91^{\circ} \mathrm{F}$ |
| :---: | :---: | :---: | :---: | :---: |
| $88^{\circ} \mathrm{F}$ | $88^{\circ} \mathrm{F}$ | $90^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $81^{\circ} \mathrm{F}$ |
| $71^{\circ} \mathrm{F}$ | $80^{\circ} \mathrm{F}$ | $79^{\circ} \mathrm{F}$ | $84^{\circ} \mathrm{F}$ | $96^{\circ} \mathrm{F}$ |

## Q Model It

You can make a table to understand the problem.

| Data Value | Deviation from Mean <br> Mean $=89.3^{\circ} \mathrm{F}$ | Absolute Deviation |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $99^{\circ} \mathrm{F}$ | 9.7 | 9.7 |  |  |
| $88^{\circ} \mathrm{F}$ | -1.3 | 1.3 |  |  |
| $71^{\circ} \mathrm{F}$ | -18.3 | 18.3 |  |  |
| $113^{\circ} \mathrm{F}$ | 23.7 | 23.7 |  |  |
| $88^{\circ} \mathrm{F}$ | -1.3 | 1.3 |  |  |
| $80^{\circ} \mathrm{F}$ | -9.3 | 9.3 |  |  |
| $99^{\circ} \mathrm{F}$ | 9.7 | 9.7 |  |  |
| $90^{\circ} \mathrm{F}$ | 0.7 | 0.7 |  |  |
| $79^{\circ} \mathrm{F}$ | -10.3 | 10.3 |  |  |
| $97^{\circ} \mathrm{F}$ | 7.7 | 7.7 |  |  |
| $84^{\circ} \mathrm{F}$ | -5.3 | 5.3 |  |  |
| $84^{\circ} \mathrm{F}$ | -5.3 | 5.3 |  |  |
| $91^{\circ} \mathrm{F}$ | 1.7 | 1.7 |  |  |
| $81^{\circ} \mathrm{F}$ | -8.3 | 8.3 |  |  |
| $96^{\circ} \mathrm{F}$ | 6.7 | 6.7 |  |  |
|  | MAD: $\frac{119.3}{15}=7.95$ |  |  |  |

## Mathematical Discourse

- Use your own words to describe what the mean absolute deviation (MAD) tells you about a set of data.

Listen for responses that the MAD describes how spread out the data are. A data set for which the MAD is low is one where each value is close to the mean value. If students are uncertain, use data sets from previous pages to review and discuss.

- How does using a table help you find the mean absolute deviation?

Students may say that having the data in a table keeps the data organized. They might also say that the table reminds them of the steps they must use.

## AT A GLANCE

Students revisit the problem on page 300 and discuss the significance of the Mean Average Deviation.

## STEP BY STEP

- Read Connect It as a class. Be sure to point out that the questions refer to the problem on page 300.
- As you talk about the mean average deviation, stress that deviation refers to how spread-out the data are.
- Have a volunteer explain the difference between high variability and low variability. Discuss why outliers increase the variability.
- Make sure students understand that the mean measures the center of the data, while the MAD measures how spread out the numbers are.


## Concept Extension

## Apply the concept of mean absolute deviation to real-world situations.

- Have students visualize two bags of potatoes. The mean length of the potatoes in each bag is $4 \frac{1}{2}$ inches. The MAD for one bag is 2 ; the MAD for the other bag is $\frac{1}{2}$.
- Discuss what the MAD means in the context of the potatoes. You want to buy potatoes to make mashed potatoes. Ask if the MAD should influence your decision as to which bag to buy.
- Have students picture two boxes of square tiles. The mean length of the tiles in each box is $4 \frac{1}{2}$ inches. The MAD for one box is 2 ; the MAD for the other box is $\frac{1}{2}$.
- Discuss what the MAD means in relation to the tiles. You want to buy tiles to re-tile your shower. Ask if the MAD should influence your decision as to which box to buy.
- Have students think of situations in which consistency is not important and others in which it matters greatly. Discuss why knowing the mean absolute deviation can be helpful.

8 Part 3: Guided Instruction
Lesson 29
Q. Connect It

Now you will solve the problem using the model.
7 What does the MAD mean within this context?
On average, the temperature varies by 8 degrees from the mean of $89.3^{\circ} \mathrm{F}$.
8 Given this context, does the MAD value indicate a high or low variability? With 15 temperatures recorded, an average deviation of 8 degrees from the mean is a high variability. There is a pretty large temperature difference between low 80 s and high 90 s.

9 If you replace the outlier $\left(113^{\circ} \mathrm{F}\right)$ with $100^{\circ} \mathrm{F}$, what happens to the MAD? Explain. The MAD decreases from 7.95 to 7.09 , because the sum of the absolute distances from the mean decreases.

10 Within this context, how are the mean and the MAD different? How are they related? The mean is a measure of center; if all the data values were equally
distributed, the temperature would be $89.3^{\circ} \mathrm{F}$ for 15 days. The MAD is a measure of variability; any temperature is an average distance of 8 degrees away from the mean.
, Trylt
Use what you just learned about mean and MAD to answer this question
11 Are the mean and MAD typically affected by outliers? Explain.
Yes. The mean changes when high or low outliers are removed. Removing the high outlier in this problem decreased the mean. Changing the mean always changes the MAD because to find the MAD, you find the distances of each point from the mean.

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## TRY IT SOLUTION

11 Solution: Yes. The mean changes when high or low outliers are removed. Removing the high outlier in this problem decreased the mean. Changing the mean always changes the MAD because to find the MAD, you find the distances of each point from the mean.

ERROR ALERT: Students who answered "no" may have confused mean with median.


## AT A GLANCE

Students solve problems using measures of center and variability.

## STEP BY STEP

- Ask students to solve the problems individually using graphs and tables when possible.
- When students have completed each problem, have them Pair/Share to discuss their solutions with a partner or in a group.

38 Part 4: Guided Practice
Lesson 29

13 Complete the table below. How many data points are represented in this histogram?


| Intervals (seconds) | Number of Drivers |
| :---: | :---: |
| $0-0.1$ | 0 |
| $0.11-0.2$ | 0 |
| $0.21-0.3$ | 1 |
| $0.31-0.4$ | 2 |
| $0.41-0.5$ | 7 |

14 Look at the histogram in problem 13. Which of the following statements do you know to be TRUE?
A The data are skewed.
B The median is 0.55 seconds.
C The range is 0.7 seconds.
D The center of the data is between 0.5 to 0.6 seconds. Jo chose $\mathbf{B}$ as the correct answer. How did she get that answer? She thinks the median is the middle of the interval in the center.
-Pair/Share In what interval are most of the data values clustered? Does this make sense given the context?
How many data points
are in each interval?

How do histograms show center and variability?


OPair/Share How can you help Jo understand her error?

## SOLUTIONS

Ex An ordered list of the numbers is shown to demonstrate that most weights are around 50 g . The mean is not as good of a measure because 20.5 is an outlier. The median is the better measure of center.
12 Solution: Without the outlier, the mean and median are both good measures of center; Students could solve the problem by finding that both the mean and median are about 50.2. (DOK 2)

13 Solution: 36; Students could solve the problem by finding and adding the frequency of each interval. (DOK 1)

14 Solution: D; Jo assumed the median is the middle of the center interval.
Explain to students why the other two answer choices are not correct:
$\mathbf{A}$ is not correct because the data clusters around the center.

C is not correct because the exact maximum and minimum values cannot be determined from the histogram. (DOK 3)


## AT A GLANCE

Students use graphs and measures of center and variability to solve problems that might appear on a mathematics test.

## SOLUTIONS

1 Solution: A; Adding more values that are greater than the median of Class 2 would increase its median. (DOK 2)
2 Solution: $\frac{1}{3}$; Divide 120 by 320 .
$\frac{1}{2}$; Divide 160 by 320 .
$\frac{3}{4}$; Divide 240 by 320. (DOK 1)

8 Part 5: Common Core Practice

3
This table represents daily attendance at 2 movie theaters for one week.

|  | Theater 1 | Theater 2 |
| :--- | :---: | :---: |
| Monday | 42 | 24 |
| Tuesday | 50 | 28 |
| Wednesday | 48 | 20 |
| Thursday | 60 | 92 |
| Friday | 80 | 88 |
| Saturday | 212 | 95 |
| Sunday | 65 | 90 |

Part A
Calculate the mean and median for each theater's attendance. Round your answers to the
nearest whole number.
Theater 1: Mean $\quad 80$ Median $\quad 60$
Theater 2: Mean $\quad 62 \quad$ Median $\quad 88$
Part B
Which is a better measure of center for Theater 1, mean or median? Explain.
Median. The mean is too high because of an outlier, 212. Almost all of the data
points are between 40-80, so the median, 60 , is a better measure of center.

Part C
Which is a better measure of center for Theater 2, mean or median? Explain.
Mean. 3 days had low attendance and 4 days had high attendance. The median
doesn't reflect the 3 days of low attendance.

Self Check Go back and see what you can check off on the Self Check on page 265.


## 3 Part A Solution:

Theater 1: Mean: 80; Median: 60
Theater 2: Mean: 62; Median: 88
Part B Solution: median; See student book page above for possible student explanation.
Part C Solution: mean; See student book page above for possible student explanation. (DOK 3)

## Assessment and Remediation

- Ask students to find the mean absolute deviation of this data set: $3,7,1,8,8,4,3,1,3,2$ [2.2]
- For students who are struggling, use the chart below to guide remediation.
- After providing remediation, check students' understanding. Ask students to find the mean absolute deviation of this data set: $2,7,5,4,6,5,4,7$ [1.25]
- If a student is still having difficulty, use Ready Instruction, Level 6, Lesson 27.

| If the error is ... | Students may ... | To remediate ... |
| :---: | :--- | :--- |
| 0 | have not used the <br> absolute value of the <br> differences from the <br> mean. | Remind students to use the absolute values of the difference <br> because distance from the mean, not direction, is important. |
| 2 | have used the median <br> instead of the mean. | Review that mean absolute deviation is based on the difference <br> from the mean value, not the median value. |
| 22 | have forgotten to divide. | Remind students that the mean absolute deviation is an average, <br> and the total of the differences must be divided by the number <br> of values. |

## Hands-On Activity

## Explore the effect of spread on the mean and the median.

Materials: drawing paper, sticky dots
On a sheet of paper, have students display the following sets of data on dot plots using sticky dots.

| Graph 1 | Graph 2 | Graph 3 | Graph 4 |
| :---: | :---: | :---: | :---: |
| $5,5,5,5,5$ | $1,5,5,5,9$ | $5,5,5,9,9$ | $1,1,5,5,5$ |

Next have students find and record the mean and median for each set of data and label them below the number lines of the graphs.

Discuss the following questions:

- If one data value is increased and another data value is decreased by the same amount, how do those changes affect the mean? The median?
- If two data values are increased, how do those changes affect the mean? The median?
- If two data value are decreased, how do those changes affect the mean? The median?


## Challenge Activity

Find and compare mean average deviations.
Materials: number cubes
Have students think about the results of rolling a number cube several times. Discuss the range of outcomes and the probable mean. Then have them consider the results of rolling two number cubes several times. Finally, have them predict which trial would have the greatest mean average deviation.
Have students work in pairs to find, record, and analyze two sets of data. Direct them to roll one number cube 10 times and record the results. Have them find the mean of the results and then the mean average deviation. Then have them roll two number cubes ten times and find the mean and MAD for those results. Have students compare the results of the two trials.

As a class, discuss the results. Have students compare the results with the predictions they made at the beginning of the activity.

